2.6b Solving Inequalities

🔑 Key Ideas

Study Tip

You can solve inequalities in much the same way you solve equations. Use inverse operations to get the variable by itself.

Addition Property of Inequality

Words If you add the same number to each side of an inequality, the inequality remains true.

Algebra If $a < b$, then $a + c < b + c$.

Subtraction Property of Inequality

Words If you subtract the same number from each side of an inequality, the inequality remains true.

Algebra If $a < b$, then $a - c < b - c$.

These properties are true for $<$, $>$, $\leq$, and $\geq$.

EXAMPLE 1   **Solving Inequalities Using Addition or Subtraction**

a. **Solve $x - 5 < -3$. Graph the solution.**

	$x - 5 < -3$	Write the inequality.
Undo the subtraction. →	$+ 5 \quad + 5$	Add 5 to each side.
	$x < 2$	Simplify.

∴ The solution is $x < 2$.

$x < 2$
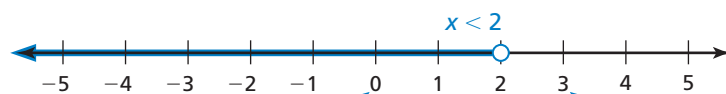
Check: $x = 0$ is a solution.   Check: $x = 3$ is *not* a solution.

Reading

The inequality $-8.3 \leq x$ is the same as $x \geq -8.3$.

b. **Solve $-3.5 \leq 4.8 + x$.**

	$-3.5 \leq \quad 4.8 + x$	Write the inequality.
Undo the addition. →	$- 4.8 \quad - 4.8$	Subtract 4.8 from each side.
	$-8.3 \leq x$	Simplify.

∴ The solution is $x \geq -8.3$.

● Practice

Solve the inequality. Graph the solution.

1. $x - 2 < 1$

2. $n + 7 \geq -4$

3. $r - 1.2 > -0.5$

4. $2.2 < 4.3 + y$

5. $\dfrac{3}{5} \geq z + \dfrac{2}{5}$

6. $m + \dfrac{1}{2} \leq -\dfrac{1}{2}$

🔑 Key Idea

Multiplication and Division Properties of Inequality (Case 1)

Words If you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Algebra If $a < b$, then $a \cdot c < b \cdot c$ for a positive number c.

If $a < b$, then $\dfrac{a}{c} < \dfrac{b}{c}$ for a positive number c.

EXAMPLE ② **Solving Inequalities Using Multiplication or Division**

a. Solve $\dfrac{x}{10} \le -2$. Graph the solution.

$$\dfrac{x}{10} \le -2 \qquad \text{Write the inequality.}$$

Undo the division. ⟶ $10 \cdot \dfrac{x}{10} \le 10 \cdot (-2) \qquad$ Multiply each side by 10.

$$x \le -20 \qquad \text{Simplify.}$$

⋮ The solution is $x \le -20$.

$x \le -20$
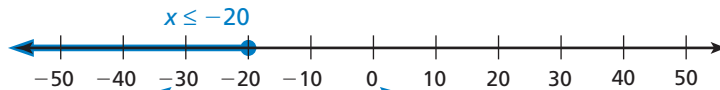
Check: $x = -30$ is a solution.   Check: $x = 0$ is *not* a solution.

b. Solve $2.5x > 11.25$. Graph the solution.

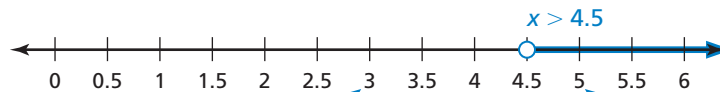$$2.5x > 11.25 \qquad \text{Write the inequality.}$$

Undo the multiplication. ⟶ $\dfrac{2.5x}{2.5} > \dfrac{11.25}{2.5} \qquad$ Divide each side by 2.5.

$$x > 4.5 \qquad \text{Simplify.}$$

⋮ The solution is $x > 4.5$.

$x > 4.5$

Check: $x = 3$ is *not* a solution.   Check: $x = 5$ is a solution.

🟢 Practice

Solve the inequality. Graph the solution.

7. $\dfrac{b}{8} \ge -5$

8. $-0.4 > \dfrac{g}{15}$

9. $\dfrac{2}{3}m \le \dfrac{8}{9}$

10. $63 < 9q$

11. $60 \le 2.4x$

12. $1.6u > -19.2$

🔑 **Key Idea**

Multiplication and Division Properties of Inequality (Case 2)

Words If you multiply or divide each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed for the inequality to remain true.

Algebra If $a < b$, then $a \cdot c > b \cdot c$ for a negative number c.

If $a < b$, then $\dfrac{a}{c} > \dfrac{b}{c}$ for a negative number c.

EXAMPLE 3 **Solving Inequalities Using Multiplication or Division**

a. Solve $\dfrac{y}{-4} > 6$. Graph the solution.

$\dfrac{y}{-4} > 6$ Write the inequality.

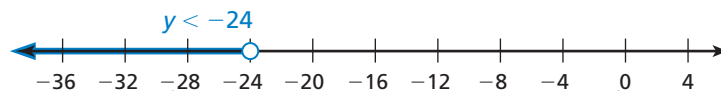Undo the division. ⟶ $-4 \cdot \dfrac{y}{-4} < -4 \cdot 6$ Multiply each side by -4. Reverse the inequality symbol.

$y < -24$ Simplify.

∴ The solution is $y < -24$.

$y < -24$

Check: $y = -28$ is a solution. Check: $y = 0$ is *not* a solution.

b. Solve $-21 \geq -1.4y$.

$-21 \geq -1.4y$ Write the inequality.

Undo the multiplication. ⟶ $\dfrac{-21}{-1.4} \leq \dfrac{-1.4y}{-1.4}$ Divide each side by -1.4. Reverse the inequality symbol.

$15 \leq y$ Simplify.

∴ The solution is $y \geq 15$.

🟢 **Practice**

Solve the inequality. Graph the solution.

13. $7 > \dfrac{j}{-1.5}$

14. $\dfrac{a}{-3} \leq -2$

15. $-2.5 < k \div (-4.8)$

16. $-2s < 24$

17. $-3.1z \geq 62$

18. $-3.9 \geq -0.6d$

EXAMPLE 4 **Solving a Two-Step Inequality**

Solve $-3x + 2 > 11$.

$$-3x + 2 > 11 \quad \text{Write the inequality.}$$

Step 1: Undo the addition. → $\quad -2 \quad -2 \quad$ Subtract 2 from each side.

$$-3x > 9 \quad \text{Simplify.}$$

Step 2: Undo the multiplication. → $\dfrac{-3x}{-3} < \dfrac{9}{-3}$ Divide each side by −3. Reverse the inequality symbol.

$$x < -3 \quad \text{Simplify.}$$

The solution is $x < -3$.

EXAMPLE 5 **Real-Life Application**

Progress Report	
Month	Pounds Lost
1	9
2	5
3	x
4	x

A contestant in a weight loss competition wants to lose at least 30 pounds in 4 months. Write and solve an inequality to find the average number x of pounds the contestant must lose in each of the last 2 months to meet the goal.

Use the progress report to write an expression for the number of pounds lost.

Pounds lost: $9 + 5 + x + x = 14 + 2x$

Because the contestant wants to lose *at least* 30 pounds, use the symbol $\geq$.

$$14 + 2x \geq 30 \quad \text{Write an inequality.}$$
$$-14 \qquad -14 \quad \text{Subtract 14 from each side.}$$
$$2x \geq 16 \quad \text{Simplify.}$$
$$\dfrac{2x}{2} \geq \dfrac{16}{2} \quad \text{Divide each side by 2.}$$
$$x \geq 8 \quad \text{Simplify.}$$

The contestant must lose an average of at least 8 pounds in each of the last 2 months to meet the goal.

Practice

Solve the inequality. Graph the solution.

19. $5n - 3 < 12$

20. $-3(w - 10) > 27$

21. $-7 \geq \dfrac{c}{-2} + 2$

22. BICYCLE You want to purchase a bicycle that costs $265. So far, you have saved $128 and you plan to save an additional $20 per week.

a. Write and solve an inequality to find the number of weeks it will take to save at least $265.

b. Graph the solution in part (a). Will you have saved enough money after 6 weeks? 8 weeks? Explain.